

# Round-Optimal Password-Protected Secret Sharing and T-PAKE in the Password-Only Model

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How to Protect a *Valuable Secret*

When all You Remember is a

*Password*

# A motivating example: Bitcoin Wallet

- Stealing Bitcoin wallets is common news: How would you protect it?
    - smartphone? lose the phone, lose the wallet; add laptop? 2 stealing targets
  - Backup in Internet server: *protection reduced to password*
    - online attacks (works for weak passwords)
    - offline server attacks: work even with reasonably secure passwords
  - Obvious cryptographic solution: keep wallet encrypted in multiple locations; *secret share the encryption key* in multiple servers
    - But how do you authenticate to the servers? With a password, of course!
      - A strong independent password with each server? Not really
      - Same (or slight-variant) password for each server? Not good
- *Each server as a single point of failure!* Didn't achieve much, did we?

# Password Protected Secret Sharing [BJSL'11]

- **Protection:** User secret shares a secret among  $n$  servers (threshold  $t$ ); forgets the secret and keeps a single password.
- **Retrieval:** User contacts  $t + 1$ , or more, servers, authenticates using the single password and reconstructs the secret.
- **Security guarantee:** Attacker that breaks into  $t$  servers and finds all their secret information (including shares, long-term keys, password file, etc.) cannot learn anything about the secret (and password).
- Only adversary hope: Guess the password, try it in an online attack.
- **Offline attacks with less than  $t+1$  corrupted servers are useless.**
- + **Soundness:** User reconstructs the correct secret or else rejects.

# PPSS: Security Definition

- As strong as possible: Only allows attacks that are *unavoidable*
- An attacker  $A$  can always test a guessed password  $p$  by one of:
  1.  $A$  interacts (as a user) with  $t+1$  servers using password  $p$ ; if  $A$ 's execution accepts then guess was correct
    - It takes *online interactions* with  $t+1$  servers to test a *single* password
  2.  $A$  simulates the sharing protocol with  $t+1$  (imaginary) servers using password  $p$  and arbitrary secret  $s$ ; then  $A$  interacts with  $U$  simulating the  $t+1$  servers. If  $U$  accepts, the guess was correct.
    - Attacker controlling  $t+1$  links to user can test a password
- Hence, if attacker controls  $t'$  servers and password chosen from  $D$ :

$$\text{Adv}_A \leq \left( q_U + \frac{q_S}{t-t'+1} \right) \cdot \frac{1}{|D|} + \epsilon$$

# More on our model

- Secure channels between user and servers assumed for initialization only (secret sharing phase)
- Reconstruction is in the CRS model (e.g., known EC group) - **no PKI or secure channels assumed**, not even between servers
  - ***user only remembers its password!***
  - Hedging property: If PKI available b/w user and servers, attack 2 is not possible (attacker advantage:  $\frac{q_U}{|D|} + \epsilon$ )
- Robustness: If U can communicate without adversarial interference with  $t+1$  servers, reconstruction succeeds (even if other links or participating servers are corrupted)

# Comparison to Prior Work

## ■ Bagherzandi-Jarecki-Saxena-Lu, CCS'11

- Formalized PPSS notion as above (roots in

All 3 protocols in ROM. We also show a 4-msg std model.

- Scheme assumes PKI between user and servers, needs 3 (or 4) messages,  $8t+7$  exponentiations for client, 16 for each server

## ■ Camenish-Lehmann- Lysyanskaya-Neven, Crypto'14:

- UC notion of PPSS (called PASS)

- no PKI b/w client and servers (except at init) , auth'd channels b/w servers

- 10 msgs,  $14t+24$  exponentiations for client,  $7t+28$  for each server

## ■ Our scheme (follows BJSJL definition)

- **No PKI**, no authenticated channels (except for initialization)

- **Single round** (2 msgs),  $2t+3$  expon's for client, 2 for e/server

# From $(t,n)$ -PPSS to $(t,n)$ -threshold PAKE

- $(t,n)$ -TPAKE: U can exchange keys securely w/ any subset of  $n$  servers using a single password as long as at most  $t$  servers are corrupted
  - exchange succeeds if undisturbed communication with  $t + 1$  servers
- We prove a Generic composition theorem: PPSS + KE  $\rightarrow$  T-PAKE.
- With the following property:
  - Single-round PPSS  $\rightarrow$  single round T-PAKE! (also w/PFS and PK KE)
- $\rightarrow$  First single-round T-PAKE:
  - no prior work achieved that, not even assuming PKI and not even for special cases such as 2-out-of-2 (ours is also the most computationally efficient)

A2



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A2 holds even with forward secrecy (Diffie-Hellman) and with single-round public-key based KE (e.g. HMQV).  
ADMINIBM, 2014/12/4



# The PPSS Scheme

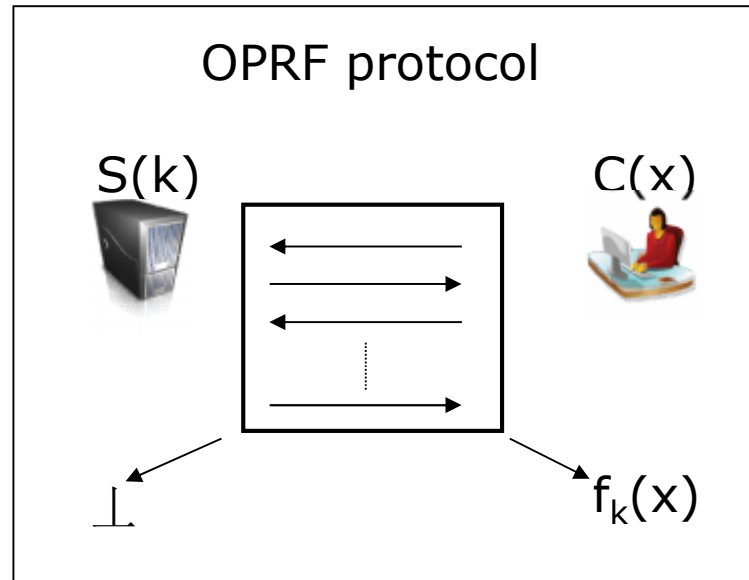
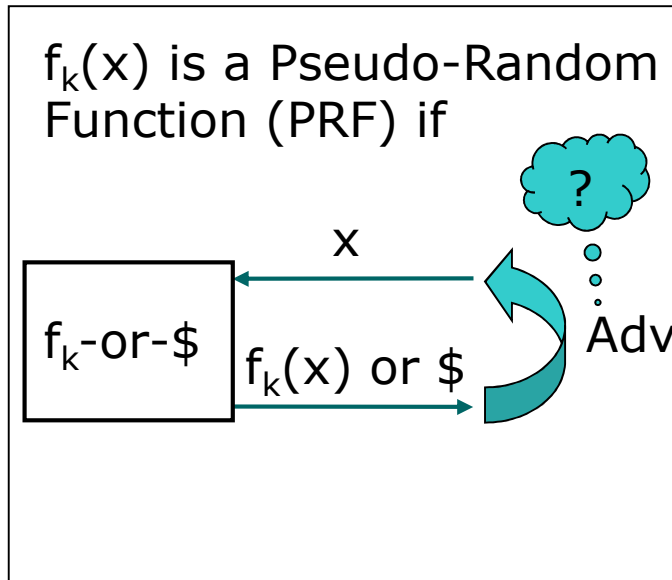


# Highlights of Our PPSS Scheme

- One round (User to Server msg + Server to User msg)
- User performs 2 exponentiations per server
  - Undisturbed communication with  $t+1$  servers suffices for reconstruction (and wrong secret never reconstructed)
- Server performs 2 exponentiations
- No inter-server communication
- No assumed PKI or secure channels (other than for initialization)

# Main building block: Oblivious PRF (OPRF)

[NR'04, FIPR'05]



- Fastest (2 exp's/party) is Hashed-DH PRF:  $f_k(x) = H(H(x)^k)$ ,
- Oblivious computation via "Blind DH Computation":  
 $C$  sends  $a = [H(x)]^r$  to  $S$ ,  $S$  replies with  $b = a^k$ ,  $C$  sets  $f_k(x) = H(b^{1/r})$ ,

# Idea of Scheme (w/o validation steps)

- Initialization: User U on password p (server  $S_i$  has OPRF key  $k_i$  )
  - Chooses random  $s$ , secret shares  $s$  into  $s_1, \dots, s_n$
  - Runs OPRF with server  $S_i$ ,  $1 \leq i \leq n$ , to obtain  $r_i = f_{k_i}(p)$ ; encrypt  $s_i$  as  $c_i = s_i \oplus r_i$
  - Stores  $c_i$  at  $S_i$ ; erases all info; memorizes p.
- Reconstruction: User U on password p
  - Receives  $c_i$  from  $S_i$  and runs OPRF to recover  $r_i = f_{k_i}(p)$ ; sets  $s_i = c_i \oplus r_i$
  - Reconstructs  $s$  from (subset of)  $s_1, \dots, s_n$
- For soundness: At initialization, U sets  $K || r = \text{PRG}(s)$ , stores  $C = \text{Commit}(pw; r)$  at each server  $S_i$ .  $K$  is defined as the secret key for reconstruction.  
  
At reconstr'n, U gets  $C$  from  $S_i$ , sets  $K || r = \text{PRG}(s)$ ; checks  $C = \text{Commit}(pw; r)$ .  
If check succeeds U outputs  $K$ , else it rejects (can use any  $C$  that  $t+1$  agree with)

# Adding Validation

- Actual protocol uses “verifiable OPRF” where user can verify correct computation of  $f_k(p)$ .
- For this, we assume  $S_i$  commits to its function  $f_{k_i}$  via a descriptor  $\pi_i$
- The commitment  $\text{Commit}(p; r)$  is augmented to  $\text{Commit}(p, \mathbf{c}, \boldsymbol{\pi}; r)$  with  $\mathbf{c} = (c_1, \dots, c_n)$ ,  $\boldsymbol{\pi} = (\pi_1, \dots, \pi_n)$ , and values  $\mathbf{c}$  and  $\boldsymbol{\pi}$  are stored at each  $S_i$
- $U$  can try reconstruction on any subset of  $t+1$  servers that agree on the values  $\mathbf{C}$ ,  $\mathbf{c}$  and  $\boldsymbol{\pi}$ . User accepts if commitment verifies correctly.
- For the DH-OPRF solution  $f_{k_i}(x) = H(H(x)^{k_i})$ , we set  $\pi_i = g^{k_i}$  and add to the protocol a DDH NIZK.
  - In progress: Relax verifiability, get rid of NIZK (except for robustness)

# PPSS Protocol (for DH OPRF)

- **Init:** Server  $S_i$  has key  $k_i$  to OPRF  $f_{k_i}(x) = H(H(x)^{k_i})$ , denote  $\pi_i = g^{k_i}$   
User  $U$  (on password  $p$  and servers' functions  $\pi_1, \dots, \pi_n$ )
  - Chooses random  $s$ , secret share  $s$  into  $s_1, \dots, s_n$ .
  - Runs OPRF with server  $S_i$  to obtain  $r_i = f_{k_i}(p)$ ; sets  $c_i = s_i \oplus r_i$ .
  - Defines  $\mathbf{c} = (c_1, \dots, c_n)$ ,  $\boldsymbol{\pi} = (\pi_1, \dots, \pi_n)$ , and  $\text{Com} = \text{Commit}(p, \mathbf{c}, \boldsymbol{\pi}; r)$  where  $K || r \leftarrow \text{PRG}(s)$ ; Stores at each server  $S_i$ :  $w = (\mathbf{c}, \boldsymbol{\pi}, \text{Com})$ .
  - $K$  is defined as the recoverable key
- **Reconstruction:** For each  $S_i$ : receive  $w_i$  from  $S_i$ ; set  $w$  to majority  $w_i$ ; run OPRF to get  $r_i = f_{k_i}(p)$  (verify using  $\pi_i$  from  $w$ ); set  $s_i = c_i \oplus r_i$ .
- Reconstruct  $s$  from  $s_i$ 's ; set  $K || r \leftarrow \text{PRG}(s)$ ; set  $C = \text{Commit}(p, \mathbf{c}, \boldsymbol{\pi}; r)$ ; reject if  $C$  differs from  $\text{Com}$  value in  $w$ , otherwise output  $K$ .

# Defining "Verifiable OPRF"

- OPRF notion is intuitive: Secure two-party computation of  $f_k(x)$  where one party holds  $k$  and one holds  $x$
- Yet, defining OPRF security is challenging:
  - E.g.: Secure 2-PC may impose input extraction, prevents concurrency, requires secure channels (all elements we want to avoid)
  - Indistinguishability definition tricky too: What's the test for the attacker after running  $q$  protocol executions (on unknown inputs)?
- We formulate a UC definition of "Verifiable OPRF" (user can check that the server uses same function consistently: e.g., always same output on pwd)
  - We bypass input extraction via ticketing mechanism
    - per-server ticket: increases with each server call, decreases with server output, no output from functionality if ticket = 0
- We show instantiations in ROM (DH, RSA), under one-more assumpt'n, and standard model (NR)



# Comparison to Prior Work (PPSS and T-PAKE)

Achieving single-round password-only protocol in the CRS and ROM models for arbitrary  $(n, t)$  parameters with no PKI requirements for any party and no inter-server communication (except for server authentication at initialization).

scheme	$(t+1, n)$	ROM/std	client	inter-server	msgs	total comm.	comp. C   S
BJKS [2]	$(2, 2)$	ROM	PKI	PKI	7	$O(1)$	$O(1)$
KMTG [6]	$(2, 2)$	Std/ROM	CRS	sec.chan.	$\geq 5$	$O(1)$	$O(1)$
CLN [4]	$(2, 2)$	Std/ROM	CRS	PKI	8	$O(1)$	$O(1)$
DRG [5]	$t < n/3$	Std	CRS	sec.chan.	$\geq 12$	$O(n^3)$	$O(1) \mid O(n^2)$
MSJ [7]	any	ROM	PKI	PKI	7	$O(n^2)$	$O(1) \mid O(n)$
BJSL [1]	any	ROM	PKI	PKI	3	$O(t)$	$8t+17 \mid 16$
CLLN [3]	any	ROM	CRS	PKI	10	$O(t^2)$	$14t \mid 24 \mid 7t \mid 28$
Our PPSS1	any	ROM	CRS	none	2	$O(t \log n)$	$2t+3 \mid 2$
Our PPSS2	any	Std	CRS	none	4	$O(kt \log n)$	$O(tl) \mid O(l)$



**Thanks!**

<http://eprint.iacr.org/2014/650>